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The stress-strain characteristics of uranium

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The stress-strain characteristics of uranium

Abstract

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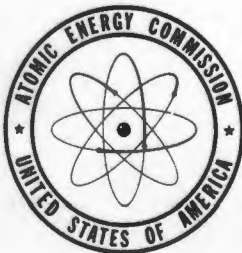
UNITED STATES ATOMIC ENERGY COMMISSION

**THE STRESS-STRAIN CHARACTERISTICS
OF URANIUM**

By
David A. McCutchan
Glenn Murphy

December 1955

**Ames Laboratory
Iowa State College
Ames, Iowa**



Technical Information Service Extension, Oak Ridge, Tenn.

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F. H. Spedding, Director of Ames Laboratory.

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TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION.....	1
REVIEW OF LITERATURE.....	1
A. The Deformation of Metals.....	1
1. Static loading.....	1
2. Cyclic loading.....	2
3. Variables affecting deformation.....	3
B. The Deformation of Uranium.....	4
1. Crystallography of deformation.....	4
2. Tensile properties.....	5
OBJECTIVES OF INVESTIGATION.....	5
A. Analytical Objective.....	5
B. Experimental Objectives.....	5
A PROPOSED MODEL OF DEFORMATION.....	8
A. Action under Static Loading.....	8
B. Action under Cyclic Loading.....	10
MATERIALS AND APPARATUS.....	18
A. Materials.....	18
B. Apparatus.....	18
PROCEDURE.....	20
A. Experimental Procedure.....	20
B. Analytical Procedure.....	21
EXPERIMENTAL RESULTS.....	22
A. Effect of Reversed Loading.....	22
B. Effect of Repeated Tensile Loading.....	22
C. Effect of Strain Rate.....	28
D. Effect of Resting.....	31
DISCUSSION OF RESULTS.....	33
LITERATURE CITED.....	36

THE STRESS-STRAIN CHARACTERISTICS
OF URANIUM*

by

David McCutchan and Glenn Murphy

ABSTRACT

Tests were made in reversed loading and in repeated tensile loading on thirteen specimens of rolled, alpha-uranium at room temperature. Constant strain rates ranging from 0.0003 in./in./min. to 0.0060 in./in./min. were employed. The data obtained lead to the following conclusions for the material tested:

1. The proportional limit for this material, if it exists, probably lies below 500 psi.
2. The elastic energy recovered per loading cycle appears to be almost entirely a function of stress; it is not altered appreciably by repeated loading or by varying strain rate.
3. The elastic energy recovered on unloading is a more representative quantity than the work of plastic deformation, as the variation for different specimens is less.
4. The work of plastic deformation in a repeated loading test decreases with the number of repetitions of load and with increasing strain rate.
5. The secant modulus from 5,000 to 15,000 psi approaches a stable value in reversed loading.
6. Resting between cycles of repeated tensile loading resulted in a partial loss of the elasticity acquired during the previous cycling.

*This report is based on an M.S. thesis by David A. McCutchan submitted December, 1955, to Iowa State College, Ames, Iowa. This work was done under contract with the Atomic Energy Commission.

These conclusions, excepting 1, are strictly applicable only for the strain rates and cycle ranges investigated. In addition, these conclusions must be qualified as valid within the limits of accuracy of the procedure. Since in fatigue failure, the failure of a microscopic volume of material may be critical, even very slight differences in succeeding loading cycles may be significant.

No definite conclusion can be drawn regarding the relative validity of the models proposed here and by Orowan. Although the models differ basically, supplementary assumptions must be supplied in both concepts in order to apply them to actual deformation. For example, time-dependence is not considered in either view. These supplementary considerations probably modify the basic mechanism to a considerable degree and render the theories difficult to verify experimentally.

INTRODUCTION

The philosophical concept of the fundamental unity of natural phenomena is fast acquiring experimental support in the study of the deformation of metals. On the atomic level the mechanisms by which metals deform in static loading, creep and fatigue are identical but their gross effect in a mechanical test depends on the many interrelated geometrical and time dependent variables characteristic of the test.

Still, phenomenological evidence of the sort obtained in a tension test may be expected to give information of a basic nature if the variables in testing are minimized. In the present work a deformation model is assumed as a foundation upon which to correlate the effects of static and cyclic loading.

The mechanical properties of uranium are of particular interest since at the time of writing the economic utilization of uranium as a solid fuel is limited by distortion of the fuel elements, which necessitates expensive reprocessing after short irradiation times.

REVIEW OF LITERATURE

A. The Deformation of Metals

1. Static loading

Houwink (1, p. 110) distinguishes the following stages in the deformation of a polycrystalline specimen by static loading:

I. Loads of such small magnitude that all crystallites plus all inter-crystalline matter are elastically strained.

II. Elastic deformations of most of the crystallites, accompanied by plastic deformations of certain parts of the material (either crystalline or inter-crystalline matter), and coupled in such a way that upon unloading the specimen practically resumes its original form. Considered in bulk, the specimen still behaves as perfectly elastic, although an elastic after-effect may be observed.

III. Elastic deformations of certain crystallites, again accompanied by plastic deformations in other parts of the material; but with the latter preponderant to such a degree that upon unloading the elastically strained crystallites have not sufficient power to bring the specimen back to its original form. Hence, a permanent set will be observable also in the exterior form of the specimen.

IV. Plastic deformations occur to such a degree that the elastic elements no longer form "through-connections" from one end of the test piece to the other of sufficient strength to withstand the load. Appreciable yielding sets in.

Although the first and second transitions appear to correspond to the proportional and elastic limits respectively, the third is indefinite.

2. Cyclic loading

Gough (2) demonstrated the identity of the processes underlying both static and fatigue failure. Plastic deformation in either case occurs by slip and twinning along the same planes and directions. Earlier Gough (3) suggested that in fatigue local strain hardening occurs, possibly leading to the formation of a crack, and that plastic strain might increase at a decreasing rate.

Jenkin (4) designed a mechanical model by which he reproduced several phenomena in the deformation of metals, such as the stress strain curve and cyclic loading effects. The model consisted of several units, each of which consisted of a spring and a friction element in series. The spring reproduces the elasticity of the metal while the friction member corresponds to plastic flow at constant stress.

Orowan (5) developed a theory of fatigue failure using the mechanical model shown in Fig. 1 which is essentially a more quantitative statement of the strain hardening concept proposed by Gough (3). As this model is similar to the one proposed in this study, it will be considered in more detail.

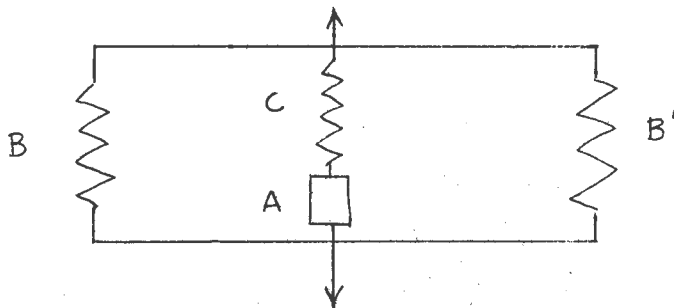


Fig. 1 Model proposed by Orowan

The springs B and B' represent the elastically strained bulk of the specimen. A represents a plastic spot, and C represents the elastic action of the plastic region and its immediate surroundings. As alternate loads are applied to the model, the plastic region strain hardens and the amplitude of the plastic strain decreases, approaching zero in a geometric progression. Thus the maximum stress that can be developed in region A is limited; if the stress at fracture is less than the maximum stress, A fails and a crack forms, if not, the system can withstand an indefinite number of cycles without failure.

The analysis described above neglects any Bauschinger effect in A, so that an element of strain hardening occurs with each stress reversal. This is in disagreement with the results of ordinary axial tensile and compressive tests, such as those performed by Sachs and Shoji (6) on brass single crystals and by Woolley (7) on polycrystalline copper. According to Barrett (8) the occurrence of a Bauschinger effect in polycrystalline metals can be ascribed to the residual stress state in the metal after the first loading cycle.

The relation of hysteresis to fatigue is discussed by Gough (3) and by Cazaud (9). In some metals an initially large loop decreases with cycling, approaching a stable form, and fracture does not occur, though the loop may have considerable area. If the applied stress range is too great, only a transient decrease in area occurs, however, succeeded by an increasing loop prior to failure.

3. Variables affecting deformation

Many investigators have sought to express the stress σ in a monotonic tensile test as a function of strain ϵ , strain rate $\dot{\epsilon}$ and absolute temperature T. Such a relation is the following, proposed by Lubahn (10);

$$\sigma = A \epsilon^M \left| \dot{\epsilon}, T \right| \quad (1)$$

where A and M are constants.

Lewis (11) has investigated the strain hardening of uranium under cyclic loading at constant strain rates and shows that equation 1 does not apply to this case.

MacGregor and Fisher (12) investigated the effect of strain rate and temperature on the true stress - true strain curves of SAE 1020 and SAE 1045 steels and of annealed brass. Their investigations indicate that the modulus of strain hardening becomes constant at approximately the maximum load, and that the value of this modulus is independent of strain rate at room temperatures. Although the work of MacGregor and Fisher is important, the strain ranges used were too great to permit the results to apply to the present study. An extensive bibliography of previous strain-rate investigations is included.

A recent paper by Fasilyev and others (13) describes experiments in which specimens of copper and tin were strained at two consecutive rates. It was found that if the transition were made at low strains to a higher or lower rate the altered curve tended to converge with the curve which would have been produced by deformation at the second strain rate alone. The curves did not converge if the transition were made at large strains, however. The authors concluded that the larger stresses obtained for a given strain at higher strain rates are due to the accumulation of excess distortions and that the stability of these distortions increase with strain, explaining the failure of the curves to converge.

This work leads to the present view that stress in a monotonic tension test is not a function only of strain, strain rate and temperature but is influenced by prior strain history.

B. The Deformation of Uranium

1. Crystallography of deformation

Uranium in the alpha phase is unusual because of its orthorhombic crystal structure, a lattice of lower symmetry than the familiar metals, which is likened by Jacob (14) to a deformed hexagonal close-packed structure. Its structure is unique, in that two of the four atoms of the unit cell tend to form covalent bonds with neighboring atoms, so that the structure is not wholly metallic. In addition, uranium exhibits anisotropy in many of its physical properties, particularly thermal expansion (15).

Cahn (15) obtained coarse-grained metal by recrystallization after a small extension. Deformation was made mechanically or by heating and cooling, making use of the anisotropy of thermal expansion. He observed slip, cross-slip, twinning and kink bands. These bands are layers in which the structure is tilted slightly about an axis in the slip plane perpendicular to the slip direction. Lloyd and Chiswik (16) have confirmed these observations with compression tests of uranium single crystals.

2. Tensile properties

In general the behavior of uranium in a tensile test is semiplastic; the tensile stress-strain curve exhibits no proportional range and marked hysteresis effects are observed on unloading and reloading.

The most complete source of data on the tensile properties of uranium in the unclassified literature is the article by Saller (17) in the section of the Reactor Handbook on general properties of materials. Saller (17, p. 393) observes that "the mechanical properties [of uranium] are affected considerably by orientation, which is controlled by fabrication history and heat treatment."

The modulus of elasticity of uranium at room temperature, determined by measuring the velocity of sound in the material, is 29,700,000 psi, the shear modulus is 12,100,000 psi and Poisson's ratio is 0.23. These properties were substantially the same for swaged, cast, and extruded specimens (17).

Table I is adapted from data presented in the Handbook. No analysis of impurities is given, although carbon content exerts a considerable influence on the mechanical properties of uranium (17). The data reported are from different lots, presumably.

Tensile properties for uranium, determined as an average value for six cycles are presented by Lewis (11). Since the same material was used in the present work, these properties are listed under Materials and Apparatus.

OBJECTIVES OF INVESTIGATION

A. Analytical Objective

The objective of the analytical investigation was to describe the deformation of a polycrystalline metal under static and repeated loads by means of a mechanical model.

B. Experimental Objectives

The purpose of the experimental work was to gain insight into the fundamental processes which govern the deformation of uranium under both static and cyclical loading, and to determine the applicability of the theoretical model to the deformation of uranium. Specifically, information was sought concerning the behavior of uranium under reversed and repeated loading, and the effects of strain rate and of resting.

Table I

Room Temperature Tensile Properties of Uranium*

Method of Fabrication, Heat Treatment	Modulus of Elasticity psi x 10 ⁶	Yield Strength 0.2% Offset psi x 10 ³	Ultimate Strength psi x 10 ³	Elongation Per Cent
as cast	24	28	56	4
rolled - 570°F., annealed - 1110°F. annealed - 1290°F.		43 25	111 64	7 9
rolled - 1110°F., annealed - 1110°F. annealed - 1290°F.		26 25	88 62	14 6
rolled - 1020°F.	25	31	96	11
rolled - 1020°F., annealed - 1560°F. quench - 1335°F. quench - 1335°F., anneal - 1020°F.	21 24 22	26 36 32	57 84 90	5 9 12

*Table adapted from the Reactor Handbook, vol. 3, section 1 (General Properties of Materials), pp. 394, 395.

Table 1 (continued)

Method of Fabrication, Heat Treatment	Modulus of Elasticity psi x 10 ⁶	Yield Strength 0.2% Offset psi x 10 ³	Ultimate Strength psi x 10 ³	Elongation Per Cent
extruded				
2:1 ratio, 390°F.	22	52	127	6
2:1 ratio, 750°F.	24	46	125	11
8:1 ratio, 750°F.	18	32	103	18
14:1 ratio, 1110°F.	16	28	96	20

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A PROPOSED MODEL OF DEFORMATION

A. Action under Static Loading

As a model of the deformation of a polycrystalline metal, the action of four equal members connected in parallel to an axial force and constructed laterally was considered. The hypothetical model is shown below:

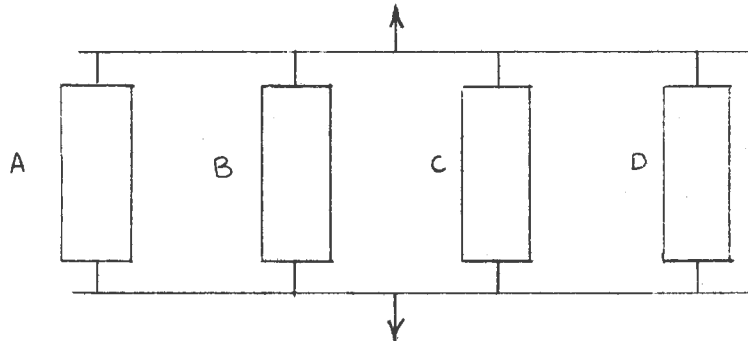


Fig. 2 Model of deformation proposed by author

A fundamental assumption in this model is that the members strain equally in response to the external load, regardless of whether they are in an elastic or plastic state. If the crystals of a metal are isotropic in regard to elasticity and the stress is uniformly distributed, an analogous equal-strain system prevails in the metal at stresses below the proportional limit. This corresponds to phase I in Houwink's description. At slightly higher stresses the elastic strengths of a number of crystals may be exceeded, but continuity of strain is preserved by the preponderantly elastic bulk of the material. The similarity thus persists to a diminishing degree until no elastic linkages remain. During succeeding deformation elastic crystals are isolated and develop a stress no higher than their plastic environment. The stress at which this occurs should be indicated by the onset of a straight line in the stress strain curve if the moduli of strain hardening of the individual crystals do not vary.

Fig. 3 illustrates the behavior of the model if the components have the same modulus of elasticity E ; elastic strengths σ_A , σ_B , σ_C and σ_D ; and the same modulus of strain hardening M .

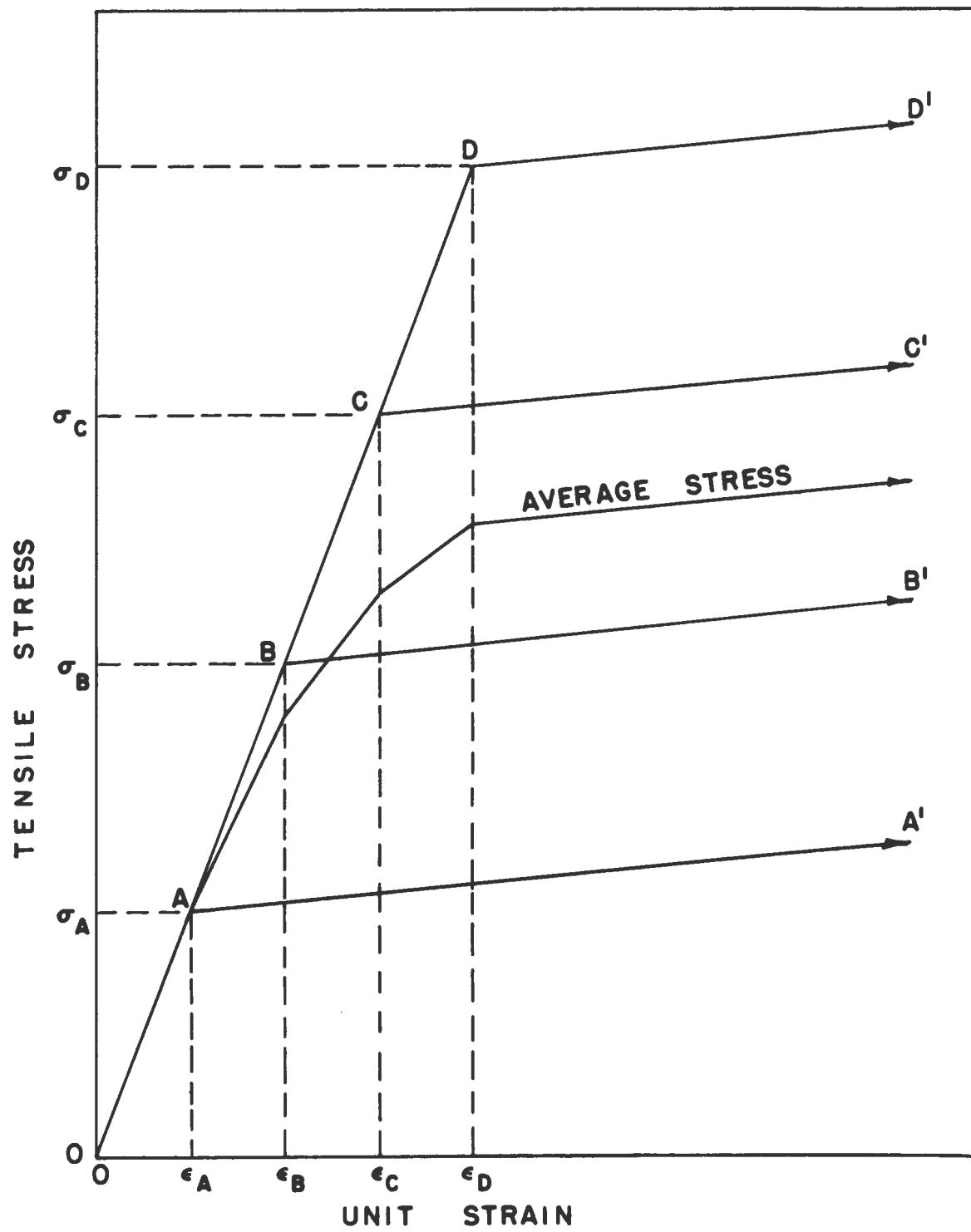


Fig. 3 Stress-strain curve for proposed model

Each element is assumed to deform elastically along line OD until its elastic strength is reached, then flow plastically as it strain hardens. The heavy line represents the average stress, calculated as the load divided by the total cross-sectional area of the elements.

As the model is strained the stress rises elastically along line OD until at point A the elastic strength of the weakest element is reached and it begins to flow plastically. Simultaneously the average stress-strain curve is deflected so that the proportional limit of the system as a whole occurs at point A.

As the applied strain increases, the stress in the elastic elements increases along line OD while the stress in the element whose elastic strength was exceeded follows the line AA'. At point B the second element enters the plastic region and the average stress-strain curve is again deflected. Likewise the remaining two elements begin to flow at points C and D.

Beyond ϵ_D the average stress-strain curve becomes linear and has a slope equal to the modulus of strain hardening of the components, M. The model is seen to exhibit a stress-strain curve similar to that of a polycrystalline metal. The shape of the curve can be altered by assuming the elastic strengths of the elements to be distributed differently with respect to strength.

B. Action under Cyclic Loading

Fatigue is generally of interest in stress ranges where no appreciable plastic deformation occurs on first loading; since the material is essentially elastic in these ranges, continuity of strain seems a reasonable assumption. Hence, an equal-strain system such as that described above is adapted to the consideration of cyclic loading in a polycrystalline metal.

The action of a two-element, equal-strain model, similar to that shown in Fig. 2, under completely reversed axial straining will be considered. The following assumptions are made regarding the model:

1. The maximum tensile and compressive strains imposed in each cycle are constant and equal.
2. The modulus of strain-hardening is constant.

3. Hysteresis and elastic after-effects in the individual elements are negligible.

4. The elastic strengths of the members do not change as a result of cycling.

5. The elastic strengths of the members are the same for tension and compression.

6. The elastic strength of the strongest element is not exceeded.

Assumption 1 reveals a fundamental difference between the model proposed here and that postulated by Orowan (4). In Orowan's model the load is applied to the plastic element by means of an elastic member in series with it; this permits cyclic strain in the plastic member to decrease as it strain hardens and is essential to the development.

Again, assumption 4 differs from the approach of Orowan, who assumed a constantly increasing resistance to plastic deformation in the plastic element. The other assumptions introduce no material differences in principle.

Fig. 4 illustrates the action of an equal-strain model loaded alternately to equal strains, $+\epsilon_A$ and $-\epsilon_A$, in tension and compression respectively. For simplicity the number of elements has been reduced to two, one of which remains in the elastic range, as specified in assumption 6.

Throughout the cycle the stress in the elastic member changes reversibly with strain, describing the line AA', whose slope is the modulus of elasticity for the material. As the model is strained in tension the weaker element deforms elastically until its elastic limit is reached at B. As the tensile strain increases to $+\epsilon_A$, the stress in the elastic member rises to A, while the weaker element flows plastically to C, increasing slightly in stress because of strain hardening.

If the load is now removed, the stress in the weaker element decreases along line CDE. Although the stress in the weaker element is zero at D, a tensile stress remains in the elastic element, so that the model contracts further until the residual tensile stress in the elastic element is equal to the compressive stress induced in the weaker element. Finally, when the external load is zero, equilibrium is attained at strain ϵ , which is the permanent deformation produced by a single tensile loading.

As the figure indicates, the elastic limit in compression for the weaker member is exceeded at E, and plastic flow in compression begins before equilibrium is reached. Whether flow in the weaker element precedes or follows the equilibrium condition depends on the geometry of the figure. Compressive straining produces further plastic flow in the weaker member and simultaneous elastic compression in the elastic member until at $-\epsilon_A$, points G and A' are reached by these elements, respectively. If the model is now unloaded, plastic flow in the weaker element again precedes equilibrium, which is reached at $-\epsilon_2$.

If the model is again loaded in tension, additional plastic flow occurs in the weaker element in the interval HJ. The maximum tensile stress in the weak element has increased by an increment σ_1 . Repeating the entire cycle will result in a second increment σ_2 , and so on.

The dashed line in Fig. 4 represents the average stress in the model, whose ordinate is the mean of the stresses in the two members, and whose abscissa is the common strain in the system. During the first tensile loading the average stress follows line OB and is then deflected at B when flow begins in the weaker member. Thus, the initial proportional limit of the system is the stress at point B. The average stress reaches a maximum at R and then descends along RS during unloading. At S it is again deflected as compressive flow begins in the weaker element. At the equilibrium strain ϵ_1 , the average stress becomes zero.

During the remainder of the cycle the average stress decreases to a maximum negative value at T, then increases and is again deflected at U. The average stress reaches zero at $-\epsilon_2$ and then rises to V as the specimen is again loaded in tension.

The fact that the average stress-strain curve was deflected at S, during the unloading from the first tension cycle, constitutes an elastic after-effect. This phenomenon can be seen in the stress-strain curve for polycrystalline uranium, Fig. 8.

If different values of the elastic limit of the weaker member are chosen, the average stress at which compressive flow in the weak member begins may be either a tensile or compressive stress. In the latter case the average stress strain curve will be deflected at some stress less than $-\sigma_1$ in absolute value. This lowering of the elastic limit in compression by

means of previous tensile loading is called the Bauschinger effect in reference to metals which have yield points. Since uranium exhibits no proportional range the term is not strictly applicable to this metal, but a corollary effect in uranium, the decrease in the initial slope of the compression curve, is observed.

As mentioned earlier, the maximum stress attained by the weaker element is increased by successive increments σ_1 , σ_2 , and so on. The following relation can be deduced from the geometry of the figure:

$$\sigma_1 = \beta(1 - \beta)^2 (\sigma_2 - \sigma_1), \quad (2)$$

where β is the ratio of the modulus of strain hardening to the modulus of elasticity. Each succeeding increment is diminished by the factor β^2 :

$$\sigma_2 = \sigma_1 \beta^2$$

$$\sigma_3 = \sigma_2 \beta^2 = \sigma_1 \beta^4$$

, and so on.

The total increase in tensile stress for the weak element in n cycles, Δ_n , is the sum of n such increments:

$$\begin{aligned} \Delta_n &= \sigma_1 + \sigma_2 + \sigma_3 + \dots + \sigma_n \\ &= \sigma_1 (1 + \beta^2 + \beta^4 + \beta^6 + \dots + \beta^{2n-2}) \\ &= \sigma_1 \left(\frac{\beta^{2n} - 1}{\beta^2 - 1} \right). \end{aligned} \quad (3)$$

Substituting equation 2 for σ_1 in equation 3 gives

$$\Delta_n = \beta(1 - \beta) \left(\frac{\beta^{2n} - 1}{\beta^2 - 1} \right) (\sigma_2 - \sigma_1). \quad (4)$$

Since $\beta^2 < 1$ the series in equation 3 converges as n approaches infinity, and the limiting increase in maximum tensile stress

for the weaker element, Δ_{∞} , is given by the relation

$$\Delta_{\infty} = \beta \left(\frac{1-\beta}{1+\beta} \right) (\sigma_2 - \sigma_1). \quad (5)$$

The maximum compressive stress reached by the weaker element simultaneously decreases. The total reduction K in the maximum compressive stress in n cycles and in an infinite number of cycles are given by equations 6 and 7.

$$K_n = \beta^2 (1-\beta) \left(\frac{\beta^{2n}-1}{\beta^2-1} \right) (\sigma_2 - \sigma_1) \quad (6)$$

$$K_{\infty} = \beta^2 \left(\frac{1-\beta}{1+\beta} \right) (\sigma_2 - \sigma_1) \quad (7)$$

The accompanying differences in the average stress are equal to one-half the corresponding differences in equations 4, 5, 6 and 7. The system as a whole becomes hardened in tension and softened to a lesser degree in compression. If the initial loading had been in compression, the reverse would be true, as can be seen by inverting the diagram.

A possible advantage of this model over the model of Orowan (5) is that the development of a stable hysteresis loop in reversed loading follows from the nature of the model and does not require the additional assumption of thermal softening which is necessary to the application of Orowan's model to this phenomenon.

If the elastic limit of the weaker element in the two-element system described above is allowed to vary, pronounced changes occur in the behavior of the system. Fig. 5 indicates the stress-strain diagram for the model if the elastic limit of the weaker element increases (solid line) or decreases (dashed line).

In the first case the loop narrows rapidly, converging toward the line described by the elastic specimen as a limiting form. A stable loop is evidently impossible in this case. If, on the other hand, the elastic limit of the weaker element decreases continually, the weaker element approaches a purely plastic situation, the average loop leaning toward the strain axis. This occurrence of this behavior in a polycrystalline metal is described by Cazaud (9). Since a negative elastic strength lacks physical

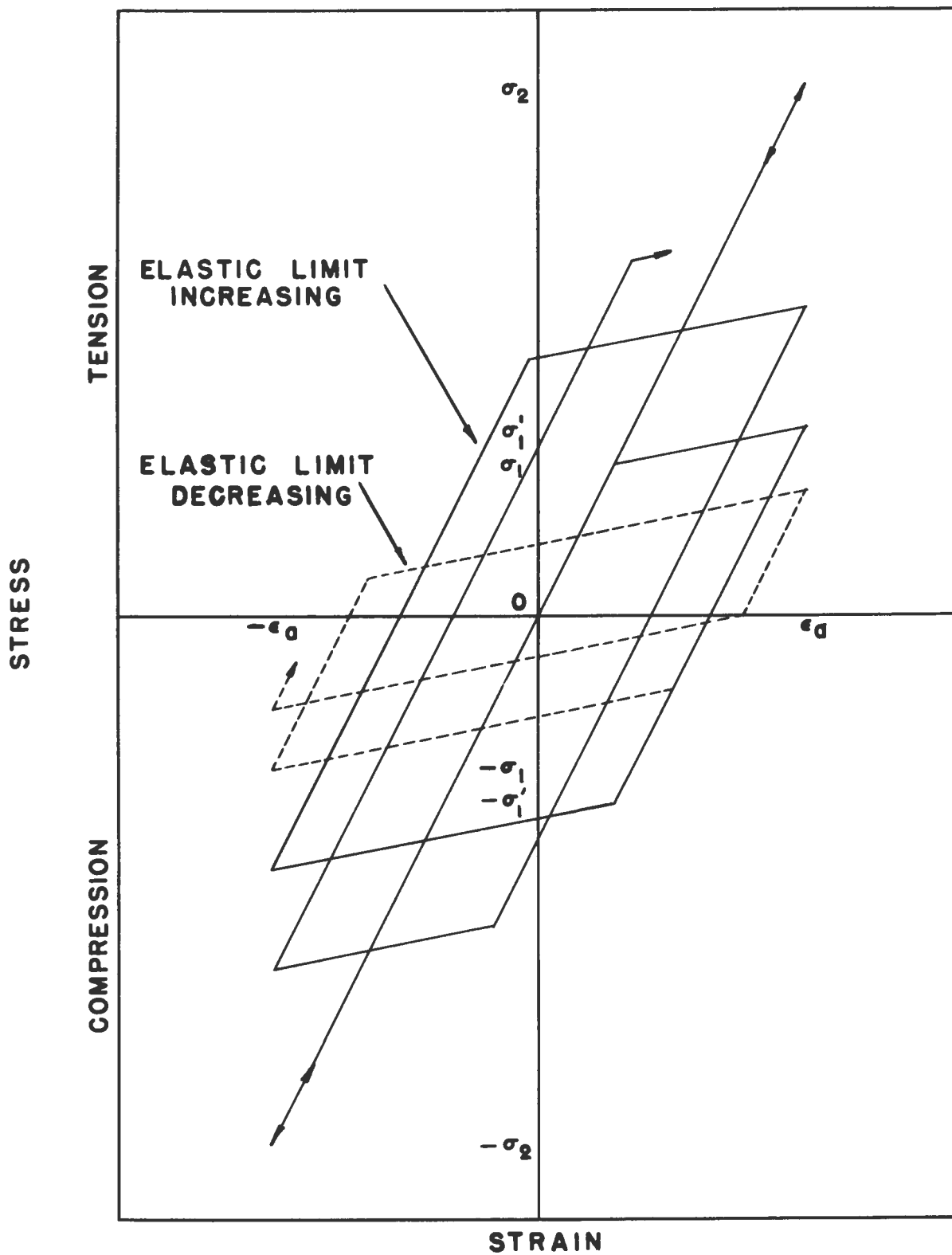


Fig. 5 Action of proposed model in reversed loading:
effect of varying elastic limits

significance the decreasing elastic limit may be assumed to approach zero, after which a geometric approach to stability occurs as illustrated in Fig. 4, where a constant elastic limit in the weaker member was assumed.

Slight changes in the elastic limit of the weaker member may thus alter the behavior of the model tremendously. In the cases where the elastic limit is constant or gradually increases the delayed fracture characteristics of fatigue will occur when the maximum tensile stress in the weaker element has risen to the fracture value, if this value lies within the limiting increase of stress. Fracture does not occur in this case if the fracture stress is greater than the limiting increase of stress.

In all of the preceding discussion the model was assumed to be brought to equal strains in tension and compression. If, on the other hand, the model is brought to the same total load (and average stress) in each sense the increasing plasticity of the weaker member and corresponding decrease of the average stress at a given strain must be compensated by increasing strains in order to support the load. In this case fracture may occur even if the elastic limit of the weaker element decreases, but could not occur if the yield point is constant, or increases continuously. The model's behavior under repeated loading, not illustrated here, reveals the presence of a hysteresis loop and a progressive reduction of loop area with number of loading cycles.

In summary, a model whose elements are strained equally in response to external loads exhibits certain phenomena characteristic of the deformation of polycrystalline metals in static and cyclic loading, if the elastic strength of its stronger members is not exceeded. These phenomena include the stress-strain curve, the elastic after-effect, hysteresis, the reduction of loop area with repeated loading, the Bauschinger effect, and fatigue failure. The analysis is similar to that followed by Orowan in his widely known "Theory of Fatigue", but differs in several important assumptions.

No attempt has been made to correlate the action of specific parts of the model to the behavior of the structural elements in a polycrystalline metal which operate in deformation. Moreover, the analysis has been treated as a problem in statics; the time-dependence of plastic flow may dominate the process of deformation in cases where the rate of straining is considerable, and the incorporation of this relationship in the action of the model may provide valuable insight into the mechanisms of deformation in metals.

MATERIALS AND APPARATUS

A. Materials

The material used in this investigation was alpha uranium in the form of slugs 1.4 inches in diameter which were fabricated from rolled rod and heat treated in the beta range. This material is described by the supplier as having less than 0.1 per cent of the impurities C, Cl, Cr, Si, B, Mg, Mn, Ni and N.

Lewis (11) gives tensile properties for nine specimens of this material, from which the following data were obtained:

Modulus of Elasticity (average value for six tension cycles):	21,800,000 psi
Yield Strength at 0.1% offset:	33,500 psi
Ultimate Strength:	91,200 psi
Reduction of Area:	9 per cent
Elongation in 1 inch:	8 per cent

This material has an endurance in rotating bending (500,000,000 revolutions) of about 19,000 psi. The data show considerable scatter, and it is impossible to ascertain if the endurance limit for an infinite number of cycles has been developed at this number of cycles.

The test specimens were obtained by sawing the slug into four longitudinal quarters, sawing these quarters laterally, and turning to size. The specimens were 2 3/8 inches long and 3/8 inch in diameter on the ends, which were threaded with 16 threads per inch. The reduced section of the specimen was 0.250 inches in diameter and 1 1/2 inches long. The proportions of this specimen are substantially those recommended by the A.S.T.M. for compression tests, so that the specimen was adapted to reversed axial loading.

B. Apparatus

A 60,000-lb. Baldwin-Southwark universal testing machine with a Tate-Emery load indicator was used in the tests. Only the lowest load range was used, for which the manufacturer's estimate of accuracy is 0.2 per cent of full scale, or 12 pounds.

The machine was equipped with a microformer automatic load-strain recorder for use with the Microformer extensometer, an inductance strain gage with a one-inch gage length and a multiplication ratio of 1000:1. The smallest strain division on the recorder chart represented 0.0001 inches, and strain increments of one tenth this value could be estimated with accuracy.

Resistance (SR-4) strain gages were applied to several of the specimens to provide more accurate strain measurements. Using a Baldwin strain recorder strain increments of 10 microinches could be read. At strain rates greater than about 0.0005 inches per inch per minute the accuracy of the measurement is limited by the inability of the operator to read the instrument.

Two specimens were subjected to cyclic loading using both types of gages, the strain gage being mounted 90° from the gage length screws of the Microformer extensometer. For specimen B-12, the strain to the maximum load of the cycle was the same for each device, but the loading and unloading curves for the resistance strain gage data lay within the loop determined by the Microformer, and the loop area was 6.3 per cent less. The differences were greater for the second loading cycle, the loading strain measured by the SR-4 gage being 8.2 per cent larger, while the loop area was about 8 per cent less.

The differences in results are probably due largely to unequal strains around the circumference of the specimen. Initial compressive strains at stresses up to 4,000 psi were measured with a resistance strain gage on specimen 3-24 during the first tension cycle, indicating that the specimens may have had eccentric loading. The fact that this initial compressive strain was removed at a relatively low stress indicates that the eccentricity in the initial loading could not contribute all of the observed differences in strain and that these differences are largely due to the inhomogeneity of the specimen.

Since the Microformer extensometer measures the average of two diametrically opposite strains, this effect is largely compensated. A further indication that unequal plastic strains may occur is the fact that relocating the extensometer produced a considerable change in the area of the hysteresis loop in a repeated loading test.

In order to secure axial loading in compression two adapters were machined from low-carbon steel. These adapters are cylindrical, 1 7/8 inches in diameter and 1 1/2 inches in height.

A 3/8 inch hole 1 inch deep was drilled in one face and threaded to receive the specimen, while the opposite end was faced in the lathe to insure perpendicularity between this end and the axis of the specimen. The specimen was thus loaded through the threads and it was not necessary to face the ends of the specimen.

A dial gage was used to measure head movement for large plastic deformations but the elongations measured in this way were unsatisfactory, as considerable lost motion is present even at large loads, perhaps due to the wedging action of the grips. Where the specimen has begun to yield with little increase in load this effect may decrease, but no means of comparison were available.

The Microformer extensometer was found to measure compressive strains satisfactorily.

PROCEDURE

A. Experimental Procedure

All of the testing was done at room temperature and at constant strain rates. The strain rate was measured by timing the rotation of the chart drum on the Microformer recorder, the rotation being proportional to the unit strain measured by the Microformer gage. The time required for the smallest strain interval on the chart (0.0001 in./in.) to pass beneath the pen of the recorder was measured by means of a stopwatch, and the loading valve of the testing machine was adjusted accordingly.

At a strain rate of 0.0003 in./in./min., the variation in rate over successive strain intervals was less than + 10 per cent, while at the highest rate of strain, 0.0060 in./in./min., relative errors of + 20 per cent were probable. The error in the average strain rate over the entire loading or unloading cycle was somewhat less than these values. Most of the loading cycles were performed at a strain rate of 0.0003 in./in./min. to maintain accuracy.

Irregularities were observed at times in the initial parts of the loading and unloading curves; whether this was due to faulty technique or to lost motion in the mechanical linkage of the extensometer was not ascertained. The best results were obtained when the gage screws were tightened firmly before loading, then tightened again after the large initial extension or compression and periodically during the test.

Where repeated loads were applied, the specimen was loaded to the same value in each cycle, and the interval between cycles was maintained nearly constant by means of a stop watch. In the reversed loading tests, the specimen was removed from the tension grips, the cylindrical compression adapters fitted, and a spherical bearing block mounted in the stationary head of the testing machine prior to the compression cycle.

Nominal stresses and strains were used in this work. It should be noted that these are close approximations of the true stress and strain for the small strains (~ 0.01 in./in.) accumulated even under repeated loading in these tests.

B. Analytical Procedure

In general, three parameters were used to evaluate the effects of the experimental variables: (1) the energy irreversibly absorbed per cycle, (2) the energy elastically stored per cycle, and (3) a secant modulus on the load-strain curve.

The energy absorbed in a loading cycle measured in in.-lb. per in.³ of stressed material is numerically equal to the area enclosed by the stress-strain curve and the strain axis. This area was determined to an accuracy of ± 0.02 in.² by planimetering and averaging the results of several consistent measurements. This corresponds to a variation of ± 0.1 in.-lb./in.³ in the energy determined. Some subjective error could not be eliminated; however, and scatter in the data are probably due in part to this uncertainty.

The energy stored elastically in a loading cycle is likewise equal to the area of the stress-strain diagram enclosed by the unloading curve, the line representing the maximum strain and the strain axis, and was also obtained by planimetering.

As mentioned earlier, occasional irregularities were observed near the origin of the load-strain curve obtained with the Microformer recorder. In addition, uranium does not exhibit a definite proportional range. To obtain a measure of the rigidity of the specimen which was independent of these factors, a secant modulus in the load interval from 250 to 750 lb. was arbitrarily chosen. This corresponds approximately to the stress interval from 5000 to 15000 psi. The secant modulus is the slope of the secant line between these loads on the load-strain curve, the corresponding stress increment being divided by the strain interval to obtain the slope in psi. For the relatively large slopes obtained in this investigation the probable error in determining the slope was about ± 4 per cent. Table 4 shows the scatter obtained with a typical test.

EXPERIMENTAL RESULTS

Thirteen specimens in all were tested under different conditions of loading as shown in Table 2. The data presented were obtained from the initial loading cycle. Considerable differences between specimens tested under the same conditions are observed. Specimens 10, 12, and 13 illustrate this irregularity. The variation in loop area (stored energy) seems to be considerably greater than the variation in the unloading curve, reflected in the elastic energy; this is confirmed by other investigators. Specimens 13, 14 and 24 are from a second slug.

The energy parameters are not given for specimen 1 because of difficulty with the extensometer. Similarly, the values of stored and elastic energy for specimen 7 are liable to be in error. The effect of irregularities in the initial parts of the loading and unloading curves is relatively great in the small loops obtained with these specimens. At larger stresses the relative effect diminishes, as the loop area increases.

A. Effect of Reversed Loading

Table 3 illustrates the variation of the secant modulus for six reversals of stress at 40,000 psi for specimen 1. The specimen previously had been subjected to 25 cycles of reversed loading at 30,000 psi, and 3 cycles at 15,000 psi.

Reversed loading tests were also made in which two cycles of tension were followed by two of compression, and so on. Fig. 6 illustrates the stress-strain curve obtained in this manner for specimen B-8. A series of tests was made in this way on specimen B-2, and the secant moduli for this series are shown in Fig. 7.

B. Effect of Repeated Tensile Loading

Fig. 8 illustrates a typical tensile stress-strain curve for uranium for two loading cycles. The decrease in loop area shown in Fig. 8, persists for many cycles. The curved line in Fig. 9 indicates this trend. Here the ordinate is the loop area (absorbed or plastic energy) for a given cycle as a per cent of the area of the first cycle. This curve was plotted on logarithmic and semilogarithmic paper and does not obey equations of the form $A + a = c(N+b)^n$ or $A + a = ce^{mN}$, where a , b , and c are constants. The fourth differences of A with respect to N were unequal, indicating that the degree of the equation of the curve must be higher than four.

Table 2
Tensile Properties of Specimens in First Cycle

Spec. No.*	Type of Loading	Strain Rate in/in/min.	Maximum Stress psi x 10 ³	Stored Energy in-lb/in. ³	Elastic Energy in-lb/in. ³	Secant Modulus psi x 10 ⁶	Type of Strain Gage
B-1	TCTC	0.0003	14.8	----	----	----	Micro.
B-2	TTCC	0.0003	40.4	50.0	26.3	19.9	Micro.
B-3	TT...	0.0003	40.7	62.2	26.5	17.4	Micro.
B-5	TT...	0.0003	40.7	61.6	29.1	19.6	Micro.
B-6	CC...	0.0003	40.7	45.0	30.1	24.4	Micro.
B-7	CTCT	0.0003	24.0	4.1	9.1	22.3	Micro.
B-8	TTCC	0.0003	40.7	78.7	38.9	14.6	SR ₄
B-9	TT...	0.0006	56.0	288	62.4	18.6	Micro.
B-10	TT...	0.0003	55.6	224	58.2	15.9	Micro.
B-12	TT...	0.0003	56.5	259 243	55.3 64.4	18.0 15.2	Micro. SR ₄
B-13	TT...	0.0003	55.5	304	61.2	17.4	Micro.
B-14	TT...	0.0012	55.0	297	64.0	15.6	Micro.
B-24	T	0.001	90.2	----	----	18.0 23.0	Micro. SR ₄

*The prefix B is occasionally omitted in the text.
T - Tension, C - Compression

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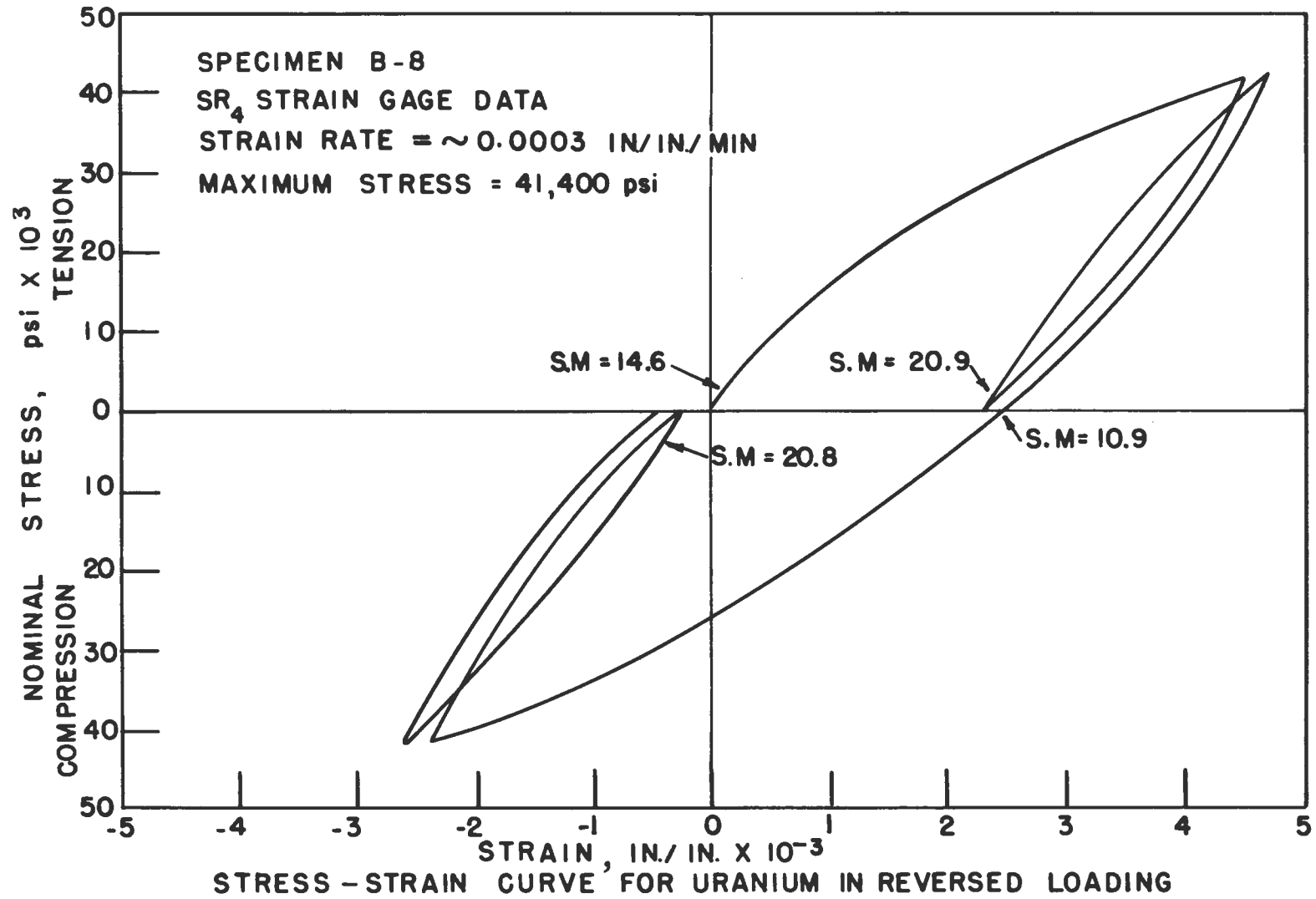


Fig. 6 Stress - strain curve for uranium in reversed loading

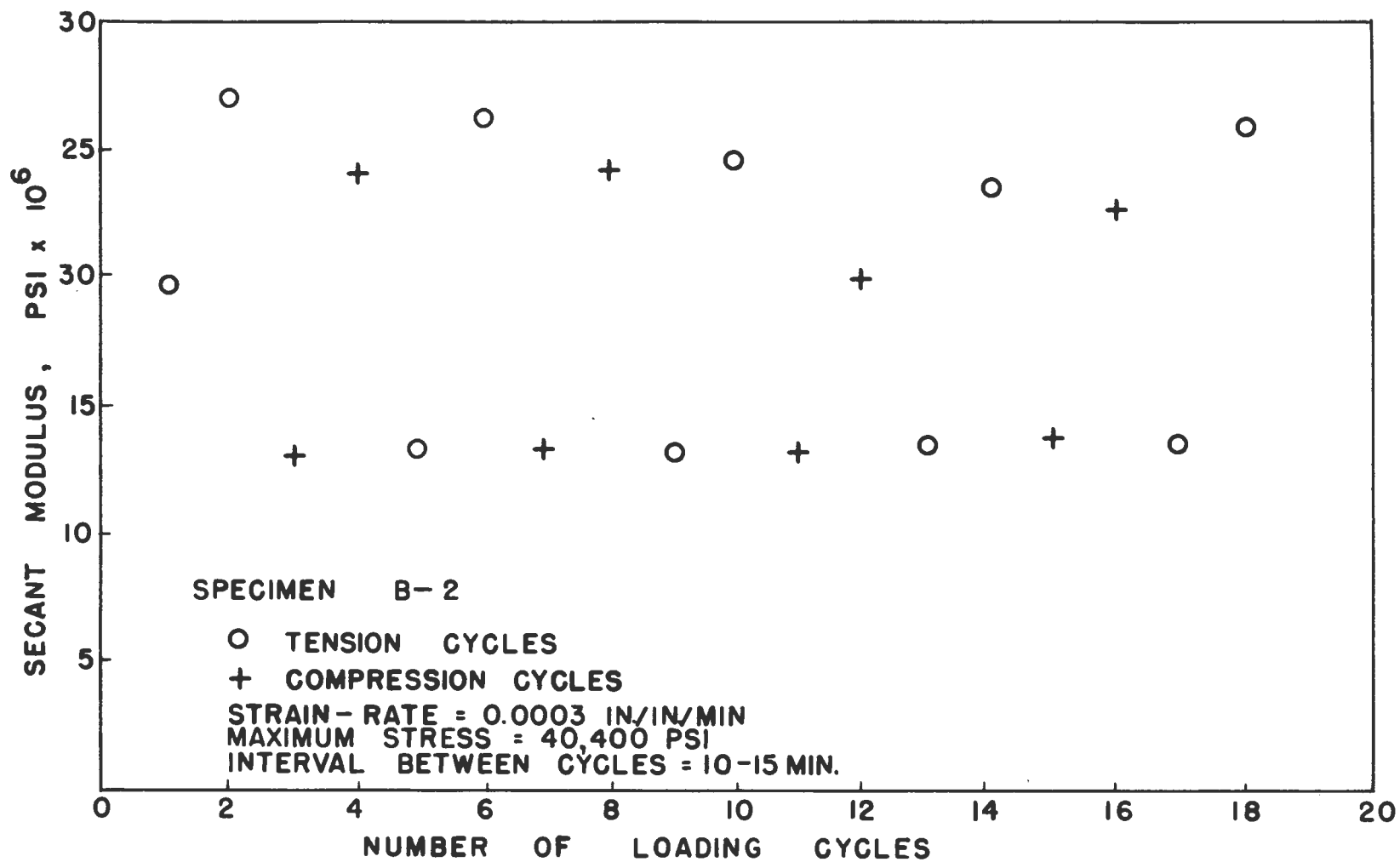


Fig. 7 Variation of secant modulus in reversed loading

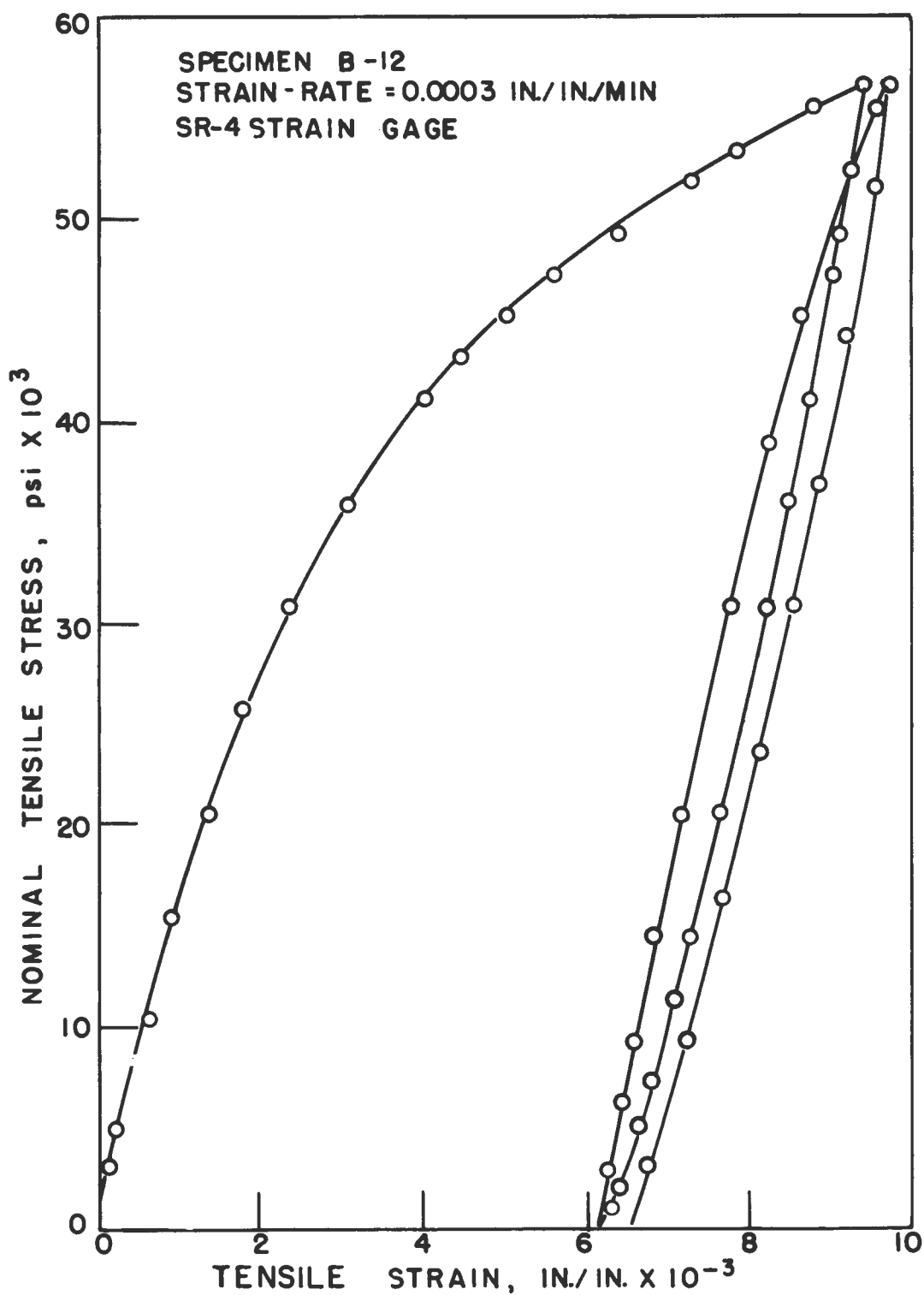


Fig. 8 Stress-strain curve of uranium
in repeated loading

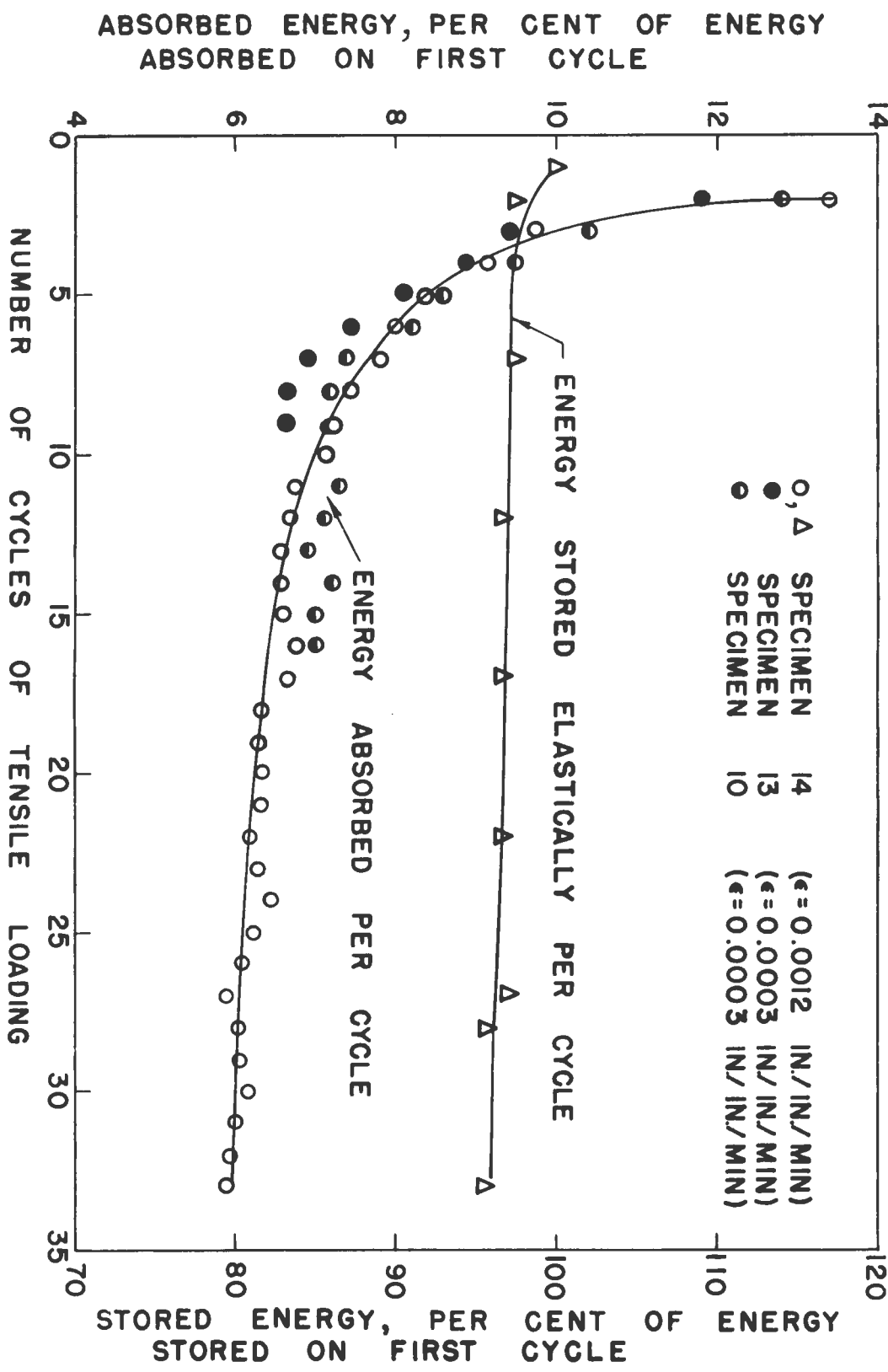


Fig. 6 Energy of elastic and plastic deformation in cyclic tensile loading

Table 3

Values of Secant Modulus Reversed Loading, $\text{psi} \times 10^6$

Type of Load	Cycle Number					
	1	2	3	4	5	6
Tension	12.3	12.3	12.1	11.4	12.0	11.9
Compression	11.6	10.5	11.6	11.5	11.6	11.2

The elastic energy stored and recovered in each cycle remains relatively constant, shown as the horizontal line in Fig. 9. The secant modulus seems to be relatively insensitive to repeated cycling. Table 4 indicates values of the secant modulus for specimen 14 over the same range of cycles illustrated in Fig. 9 for this specimen.

C. Effect of Strain Rate

Fig. 10 shows the effect of varying strain rates on the energy absorbed in each loading cycle for specimen 13. Previously the specimen had been subjected to 79 loading cycles at a strain rate of 0.0003 in./in./min. but the hysteresis loop had not attained a stable form. An approximately linear relation was assumed and straight lines were drawn for each strain rate by the method of least squares. The line is omitted for the points corresponding to 0.0040 in./in./min. due to the poor distribution of these points. The percentage difference between the average ordinate of each line and the average ordinate of the upper line in the range of cycles shown was then determined. These percentages indicate the change in the energy absorbed relative to that absorbed at the base rate of 0.0003 in./in./min.

In Fig. 11 the percentage differences in absorbed energy have been plotted against strain rate. A similar test was made on specimen 14 after 38 tensile loading cycles at 0.0012 in./in./min. Insufficient data were gathered to permit an accurate estimation of the effects of strain rate in this case; the data is shown in Fig. 11 only to confirm the general trend.

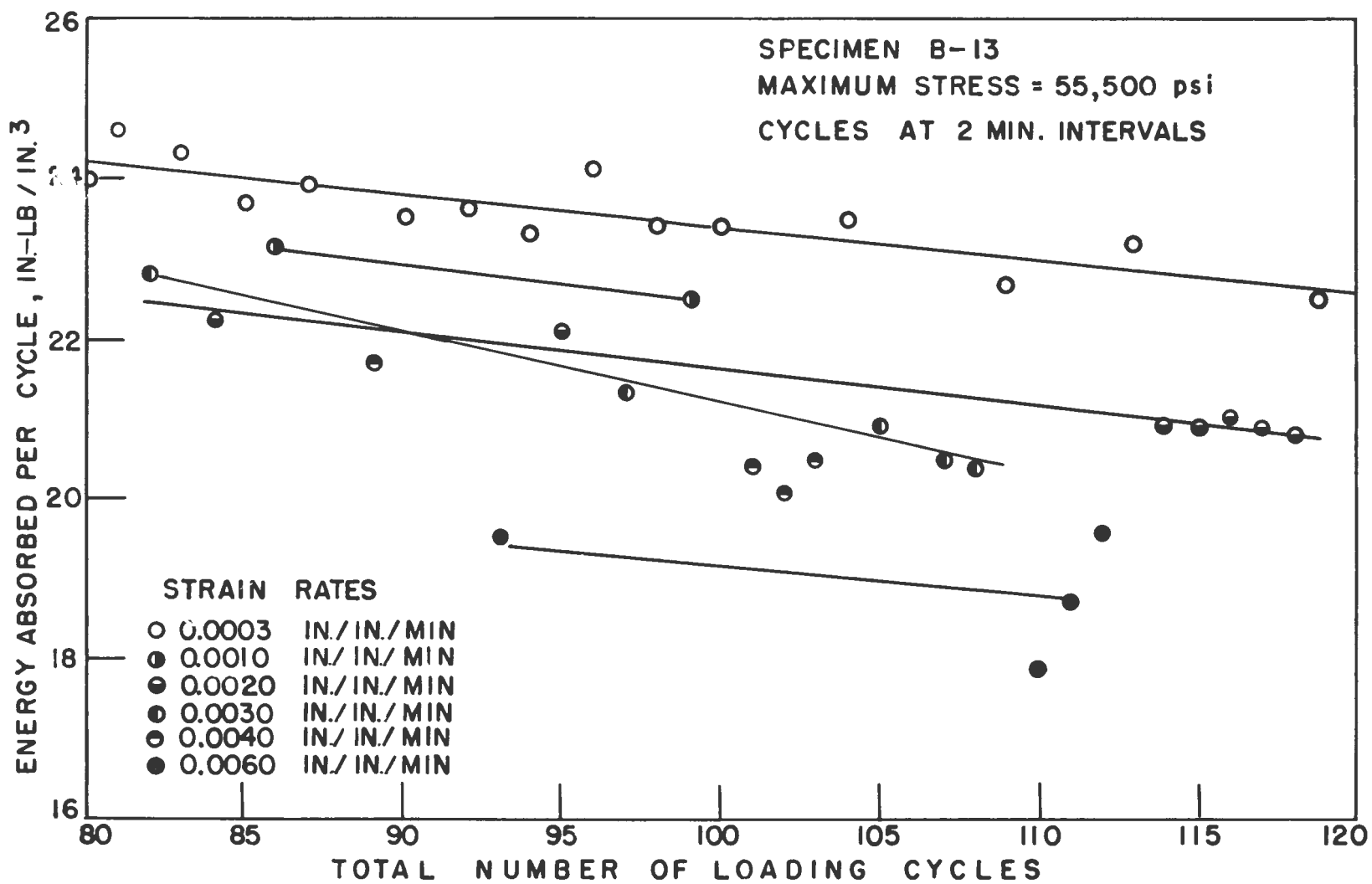


Fig. 10 Effect of strain rate on absorbed energy

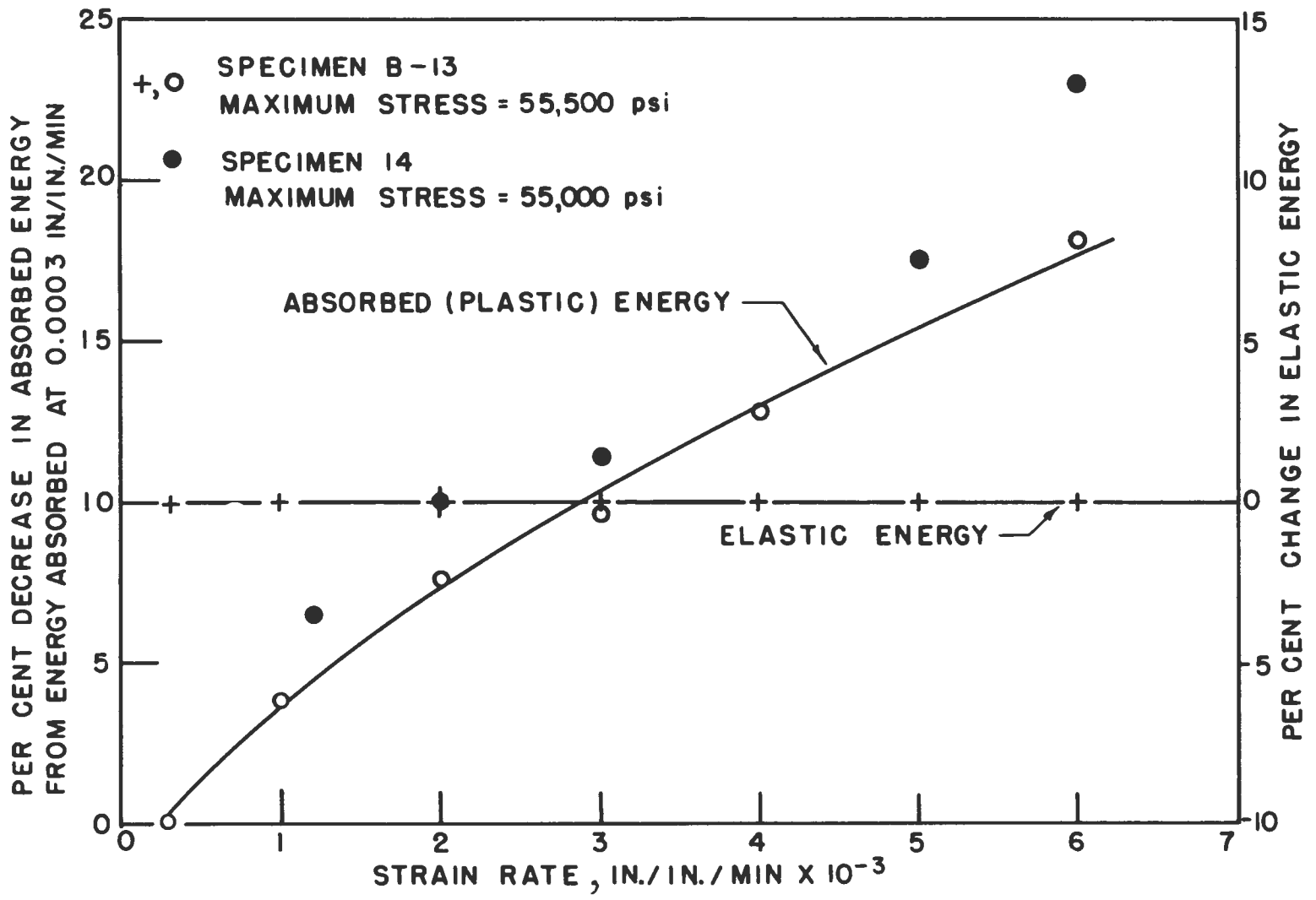


Fig. 11 Dependence of absorbed and elastic energy on strain rate

Table 4

Effect of Cyllical Tensile Loading on
the Secant Modulus for Specimen 14

Cycle Number	Secant Modulus psi x 10 ⁶	Cycle Number	Secant Modulus psi x 10 ⁶	Cycle Number	Secant Modulus psi x 10 ⁶
1	15.6	12	21.5	23	22.0
2	21.4	13	21.5	24	21.7
3	22.7	14	21.9	25	22.6
4	21.6	15	20.6	26	22.0
5	22.2	16	21.8	27	21.9
6	21.7	17	21.6	28	21.8
7	21.9	18	21.8	29	22.2
8	22.2	19	21.9	30	21.9
9	22.3	20	21.5	31	22.3
10	21.6	21	22.1	32	22.0
11	21.5	22	22.1	33	22.2
Average	21.3	Average	21.7	Average	22.1

The scatter in these data may be due entirely to the difficulty in determining the secant line, as mentioned in Section IV.

There did not appear to be any change in the elastic energy recovered per loading cycle for specimen 13, as the horizontal straight line in Fig. 11 indicates. Moreover, no change in the secant modulus of specimen 13 occurred during this test.

D. Effect of Resting

Significant increases in the loop area, indicating an increase in the energy absorbed by the specimen occur if the specimen is rested previously. Fig. 12 shows this effect for specimens 13 and 14.

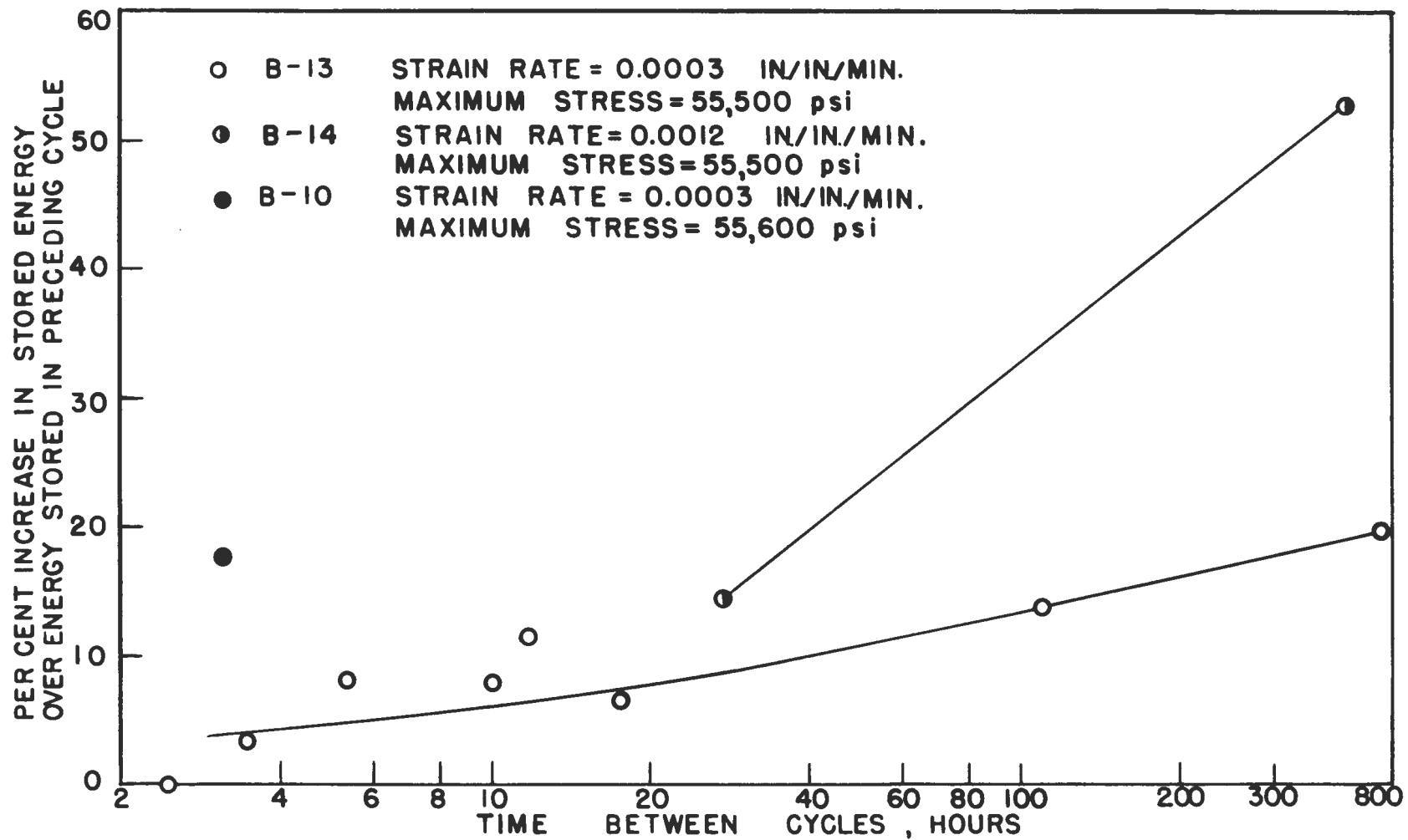


Fig. 12 Effect of resting on energy stored per cycle

DISCUSSION OF RESULTS

Comparison of the secant modulus for an initial loading in either sense, given in Table 2 with the values obtained after a loading in the opposite sense indicates a considerable softening. This behavior parallels the lowering of the yield point (Bauschinger effect) in ferrous metals.

The amount of this softening would appear to be limited according to the data in Table 3 and Fig. 7. The amount of data presented does not justify the conclusion that no further Bauschinger effect occurs; on the contrary, if the Bauschinger effect is the result of residual stresses, it may be assumed to be present until plastic flow is completed. This did not occur in the range of cycles investigated, as discussed below.

Fig. 7 demonstrates that the secant modulus approaches a constant value for the first loading in either sense. Similar results are reported by Gough (3). The large scatter in the values of the secant modulus for the second loading in each sense is probably due to the difficulty of measuring these large slopes on the load-strain diagram.

Fig. 9 indicates the gradual reduction in energy absorbed by the specimen (i.e., loop area) in repeated tensile loading to a constant maximum stress. The absorbed energy is equal to the work of plastic deformation, as no energy is irreversibly absorbed by a perfectly elastic body. The reduction in absorbed energy thus represents the increasing elasticity of the specimen, which, however, increases at a decreasing rate. Whether this decrease is a transient one followed by an increasing loop until fracture, or a gradual approach to a stable form is not known definitely, although the former view is supported by the fact that the stress level is considerably higher than the endurance limit in rotating bending.

The energy stored elastically during each cycle - that is, the energy released by the specimen during unloading - decreases only slightly after the second loading cycle, as shown in Fig. 9. The seeming contradiction with the observation that the elasticity is increasing is due to the fact that the final part of the unloading curve becomes more nearly vertical with repetitions of load. For specimen 14 the contraction on unloading in the first cycle was 0.00328 in./in., while the contraction in the second cycle was 0.00316 in./in. and in the thirty-third cycle was 0.00310 in./in. This straightening in the final part of the unloading curve, corresponding to a diminishing elastic after-effect, is thus largely completed by the second cycle. Fig. 8 illustrates this effect.

Since the work of elastic deformation remains substantially constant, the total work of deformation, which is equal to the sum of the energies of elastic and plastic deformation, decreases approximately as the latter.

At high stresses considerable plastic flow at constant load (creep) was observed, the rate of flow being at first rapid, then decreasing until no flow was perceptible after 10 min. Small variations in the delay between loading and unloading might be expected to affect the loop area considerably therefore. A 54 per cent increase in loop area was obtained in specimen 13 by maintaining the stress at its maximum value, 55,500 psi, for 10 min.

To eliminate this source of error several consecutive loadings were made in which the superior stress was maintained 10 min. so that the creep in each case would go practically to completion, but these loops exhibited even more scatter than the normal loops.

Plastic flow does not occur solely at the peak stress of the cycle: Table 4 indicates a slight increase in the secant modulus, indicating the progress of flow and strain hardening. Figures 9 and 10 illustrate the marked reduction in plastic deformation at higher strain rates, showing the time-dependence of plastic flow. The difference in area cannot be attributed entirely to creep occurring at the maximum stress. This was shown by a similar variation in area of cycles made at different strain rates but with the maximum stress maintained 10 min. for each cycle, as mentioned above.

It should be noted that the reduction in plastic deformation indicated above was determined for specimens which had been previously loaded. This relation did not hold for the first loading cycle, as can be seen from the data in Table 2. Neither the elastic energy recovered per cycle nor the slope of the secant line changed with strain rate. Fig. 10 shows the behavior of the elastic energy; the data for the secant modulus are not given.

It has been observed that the unloading curve of uranium is more representative of the material than the loading curve. This is a corollary to the fact mentioned in the section on Experimental Results that the elastic energy at a given stress is generally more nearly the same for different specimens. In general, then, the work of elastic deformation recoverable upon unloading is a function primarily of stress, and is only slightly influenced by previous stress history or strain rate.

The data in Fig. 9 indicate that the behavior in a given cycle is not affected by the strain rate in the preceding cycle. If, as Vasilyev (13) suggests, excessive distortions are induced at the faster rates, these distortions must have been dissipated in the interval between cycles.

The effect of resting between cycles was to increase the loop area visibly. Fig. 11 reveals a rapid initial loss of elasticity, this recovery of initial properties then persisting at a decreasing rate for many hours. The per cent increase in loop area appears to be roughly proportional to the logarithm of the rest time for periods longer than one day. The effect of resting varied greatly between specimens as Fig. 12 shows.

Some evidence was observed that a change in the time between cycles from 1 to 3 minutes produced significant changes in loop area. If the time between loading cycles for specimen 13 was reduced to 1 minute, the loop generally decreased in area, although the data showed considerable scatter. Because of difficulty in maintaining a constant time interval between cycles it was not ascertained if the preceding cycle represented a stable state for the rate of strain employed in the test. Therefore, it is questionable whether the effect of rest is to produce an initial reduction of area followed by increasing area, or if the area continuously increases, the reduction being due to the instability of the previous cycle. Gough (3) reports work on steel which indicates an initial softening if tested immediately after overstrain, followed by a recovery of elasticity. It would seem that an opposite effect occurs in uranium, since the effect of time is to produce a softening rather than increased elasticity.

The stress-strain data obtained with the SR-4 resistance strain gage indicate the limit of proportionality for this material to be certainly less than 2000 psi (specimen 8), and probably less than 500 psi (specimen 12).

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